

# New Approach for Fault Diagnosis Using Analytical Redundancy and Sequential Monte Carlo Methods

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## Abstract

In our days, mechatronic systems are very complex machines that require special supervision during their operation, to avoid accidents and critical faults that can be harmful to people. This paper describes a general framework to define the components of a new complete diagnostic system. The main part of this system is the diagnostic system block, where we define the decision rules and diagnostic statements to detect and isolate the faulty components. We use two approaches to diagnose dynamic systems. The methods developed are: a) Analytical redundancy and knowledge-based redundancy tools; b) The algorithms based on classical sequential Monte Carlo methods.

**Keywords:-** Fault Detection and Isolation, Particle Filtering, Fault, Diagnostic, Dynamic System.

## 1 Introduction

The development of sophisticated mechanical systems, together with the electronic systems and new edge of computers have allowed the design and production of new complex mechatronic systems. Mechatronic products are more complex, and with a greater probability of faults. Now, computer systems are able to develop a diagnostic system with the following functions: faults detection, isolation of the components with failure, and identification of the size of the fault. Additionally, an appropriate diagnostic system is necessary to take into consideration because both the machine and its environment changes with time, measurements are corrupt-

ed by noise, some quantities of interest are unobservable, and machine states are changing under different operating conditions. Another advantage of the diagnostic system is that we can reduce maintenance costs when the failures are detected on time. This paper defines a general framework for fault diagnosis, and we will test our algorithms with a mechanical system: traditional suspension system in a motor vehicle. The techniques used in the diagnostic system are:

- Analytical redundancy and knowledge-based redundancy tools. We are going to do the fault diagnosis using the structured hypothesis test, and using the transfer function obtained with Recursive Least Squares (RLS).
- Using an algorithm based on classical sequential Monte Carlo methods, such as look ahead Rao-Blackwellized Particle Filtering.

Finally, we present the experimental tests that validate the diagnostic system.

## 2 Mechanical System

The mechanical system corresponds to the suspension system of a vehicle that vibrates in the vertical direction while traveling over a road with obstacles. The suspension system was simplified at only one spring and one damper. This model is used for validation of the algorithms implemented in the diagnostic system. The parameters of the model are:

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- Mass:  $m = 1200$  kg
  - Spring constant:  $k = 400$  KN/m
  - The system presents a damper with a ratio damping  $\zeta = 0.5$ . Damper constant:  $c = 21910$  Kg/sec
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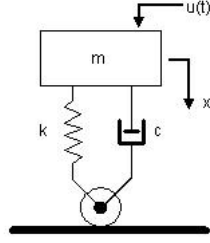


Figure 1: Vehicle suspension system model

The mechanical system can be modeled as a single degree of freedom system, as it is shown in Figure 1. Where:

$u(t)$  represents the perturbation to the system.

$m$  is the vehicle mass.

$k, c$  represents the spring and damper constants.

Applying the second law of Newton, we can define the differential equation to evaluate the motion system,

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \frac{1}{m}u(t) \quad (1)$$

The equation (1) implies two state variables:  $x_1 = x$ ,  $x_2 = \dot{x} = \dot{x}_1$ . Then, the model can be represented by the following state space:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t) \quad (2)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (3)$$

The model represented by equations (2-3) was programmed in Matlab and these equations represent the behavior of the vehicle running over a road. Suddenly, the vehicle is affected by two types of perturbations over the road. These are:

- Impulse signal.** This input will be introduced in the model at a specific time, and its magnitude will be 65000 N.
- Sinusoidal signal.** This input will be defined in agreement with the following equation, which defines the road shape.

$$u(t) = A * \sin(\omega t) = 45000 * \sin(0.2908 * V * t)$$

where  $V$  is the car's speed,  $t$  is the time,  $A$  is the amplitude, and  $\omega$  is the frequency.

### 3 General Framework for Fault Diagnosis

#### 3.1 Faults

Any deviation from the normal behavior in the process or system is considered as a fault. Figure 2 shows a diagram of the representation of a general system model with the inputs/outputs required in the evaluation of the process. One way to model a fault is

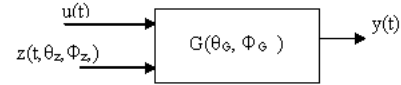


Figure 2: General system model

with the observation of the variation of the constant parameters of the system. We will be using this type to describe the faults. The constant parameters of the system are: the spring constant of stiffness ( $k$ ), and the damper constant ( $c$ ). Then, the fault vector is  $\theta_G = [k, c, m]$ .

Sometimes, it is necessary to model a fault as a variation of some signal. This type of fault modeling is represented by  $\theta_z$ . In our model, we are going to consider only faults in the parameters of the system (damper and spring)  $\theta_G$ , and we will have two types of faults: abrupt and incipient fault. For the spring, we will consider an abrupt fault, and in the damper, we will apply both types of faults.

#### 3.2 Components and faulty modes

Each system can be divided into components in order to facilitate the model construction and the fault isolation. For each component a situation of fault can occur, considering that we have a number of  $p$  components in the system, the fault state  $\theta$  of the complete system can be written as,

$$\theta = [\theta_1, \theta_2 \dots \theta_P]$$

The system has been divided in two components: the spring (S) and the damper (D). The faulty modes represent all the possible failures that can be present in the components. In our system, we have six faulty modes, and they are defined as shown in Table 1.

Table 1. Faulty modes for the system.

Faulty Mode	Component Fault Mode
$(\theta_{NF})$	$\theta_{NF} = \{[400000, 21910]\}$
$(\theta_S)$	$\theta_S = \{[k, 21910]; k = 250000\}$
$(\theta_{D1})$	$\theta_{D1} = \{[400000, c]; c = 200\}$
$(\theta_{D2})$	$\theta_{D2} = \{[400000, c]; c = 21910 - 7137 * t\}$
$(\theta_{SD1})$	$\theta_{SD1} = \{[k, c]; k = 250000, c = 200\}$
$(\theta_{SD2})$	$\theta_{SD2} = \{[k, c]; k = 250000, c = 21910 - 7137 * t\}$

The values of Table 1 define the normal situation ( $k = 400000, c = 21910$ ) and the fault situation ( $k = 250000, c = 200, orc = 21910 - 7137 * t$ ) of the components.

### 3.3 Diagnostic system

One important part of the diagnostic system is the Residual Generator block. This block receives the output and inputs of the process, and with this information it computes some quantities that indicate the inconsistency between the actual measured plant variables and the output produced by the mathematical model of the process. The structure of a fault diagnostic system implies to define some decision rules,  $\gamma(x)$ , in the Residual Generator, and some decision logic to generate a diagnostic statement,  $S = \gamma(x)$ . The decision rules tell us if the behavior of the system is derived or not from the faulty mode that contains the decision rule. Once we have the results of all decision rules, the diagnostic statement must be established. There are many ways to do this. We are going to evaluate the following options:

- A diagnostic system using an analytical redundancy methods.
- A diagnostic system using algorithms based on classical sequential Monte Carlo methods.

#### 3.3.1 Diagnostic system with analytical redundancy methods

Fault diagnosis using structured hypothesis test. This method is a generalization of the structured residuals method, [11]. It consists in the determination of the model hypothesis for each faulty mode. In our work, the hypothesis represents the median value, and the standard deviation,  $\mu_i, \sigma_i, i = (a1, a2, b1, b2)$ , of the parametric identification for each faulty mode. These values will be obtained in the Residual Generator using a RLS algorithm. The hypotheses are defined as:

$$\begin{aligned} H_0 NF &= f(\mu_{a1}, \sigma_{a1}, \mu_{a2}, \sigma_{a2}, \mu_{b1}, \sigma_{b1}, \mu_{b2}, \sigma_{b2}) \\ H_0 FS &= f(\mu_{a1}, \sigma_{a1}, \mu_{a2}, \sigma_{a2}, \mu_{b1}, \sigma_{b1}, \mu_{b2}, \sigma_{b2}) \\ H_0 FD1 &= f(\mu_{a1}, \sigma_{a1}, \mu_{a2}, \sigma_{a2}, \mu_{b1}, \sigma_{b1}, \mu_{b2}, \sigma_{b2}) \\ H_0 FD2 &= \dots \\ H_0 FSD1 &= \dots \\ H_0 FSD2 &= \dots \end{aligned}$$

The hypothesis must be tested using a test statistic. It is defined as,

$$Z_0 = \frac{\mu - \mu_0}{\sigma_0 / \sqrt{n}} = \gamma \quad (4)$$

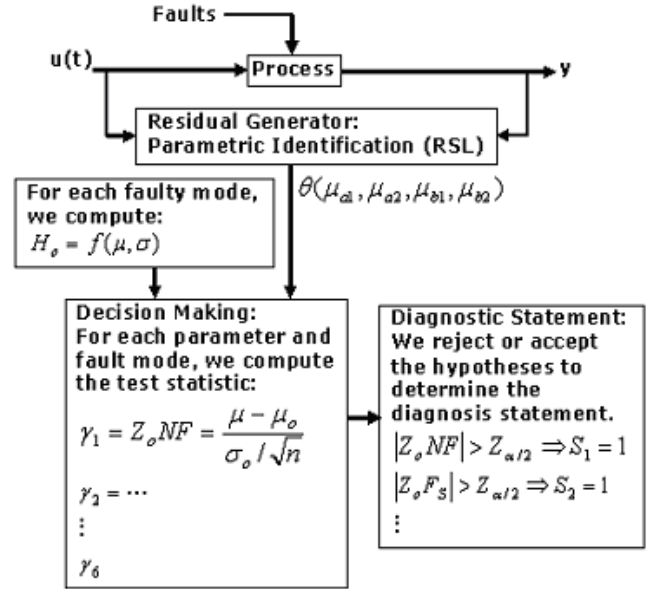


Figure 3: Diagnosis using structured hypothesis test

Where  $n$  is the number of samples. Therefore, we compute the individual test  $\gamma_1 \dots \gamma_n$  for all fault modes, and these values represent the decision rules to make the logic decisions. Taking into account that  $\gamma_k(x)$  is a hypothesis test, and a function of  $u(t)$  and  $y(t)$ , then, we need to compare its value with a standard normal distribution to accept or reject the hypothesis. We determine the diagnostic statement ( $S$ ) as follows:

$$|Z_0| > Z_{\alpha/2} \rightarrow S = 1 \quad (5)$$

The diagnostic statement ( $S = 1$ ) allows us to diagnose and isolate the faulty component with faults. Figure 3 shows the diagnostic system. The possible diagnostic statements are:

$$\begin{aligned} S_1 &= 1; \text{ Faulty system} \\ S_2 &= 1; \text{ Faulty Spring} \\ S_3 &= 1; \text{ Loss of damper} \\ S_4 &= 1; \text{ Oil leak in faulty damper} \\ S_5 &= 1; \text{ Faulty spring and loss of damper} \\ S_6 &= 1; \text{ Faulty spring and oil leak in damper} \end{aligned}$$

Fault diagnosis using the transfer function. We are using the RLS algorithm to obtain the parametric identification required in the hypothesis test diagnosis. With the parameters, we can build the transfer function that defines the behavior of the system in the discrete state space. This function has the following form,

$$H(z) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{b_1 z + b_2}{z^2 + a_1 z + a_2} \quad (6)$$

The parameters  $a_1, a_2, b_1, b_2$  are obtained by the RLS algorithm. With equation (6), we apply the classical control theory methods to make a mapped from

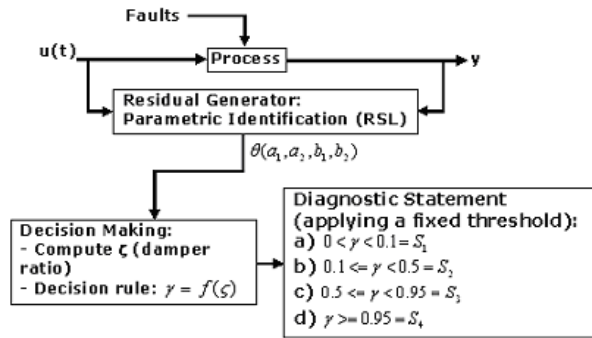


Figure 4: Diagnostic system using the transfer function

z-domain to s-domain, and then we compute the damper ratio that represent the decision rule. The flow diagram of the diagnostic system is shown in Figure 4, where the decision rules and the diagnostic statements are defined. The diagnostic statements are:

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- $S_1$  = Loss of damper
  - $S_2$  = Spring fault or oil leak in damper
  - $S_3$  = No Fault in Suspension Syst.
  - $S_4$  = Spring fault or high damping constant
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### 3.3.2 Diagnostic system using classical sequential Monte Carlo methods

Basically, we will use Particle Filters for state estimation on line. It is possible to compute the distribution of the continuous states exactly. However, it is very difficult to determine the true discrete state of the system at any point in time with certainty. We need to represent uncertainty about the state of the dynamic system, using a probability distribution over the states that system could be in. To maintain this distribution, the algorithms will perform Bayesian belief updating. The essence of the Particle Filter approach is to simulate the behavior of the system. Each sample predicts a future behavior of the system in a Monte Carlo fashion, and the samples that match the observed system behavior are kept, while that fails to predict the observations, tends to die out [8,9]. The approach described in this method is based on the observation that looking ahead at the measurements that result as a consequence of a fault can improve diagnosis. This allows us to improve the probability of having a sample following a low-probability transition because the probability of such sample is based on the posterior likelihood of the transition, rather than the probability. The method considers four steps [10]:

1. Kalman Prediction step.
2. Selection step.

3. Sequential importance sampling step.
4. Updating step.

Figure 5 shows the implemented diagnostic system for the dynamic system.

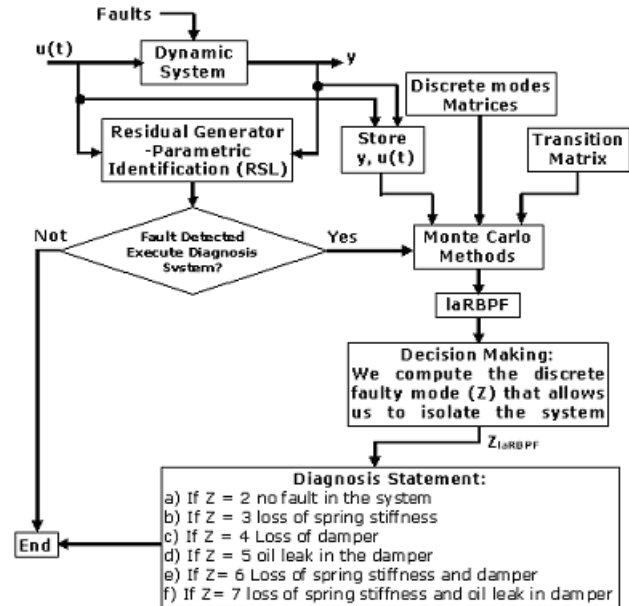


Figure 5: Diagnosis using sequential Monte Carlo methods

## 4 Proposed Tests

All the blocks were implemented in Matlab to get a simulation on line. During the simulation, we applied all the faulty modes specified for the system. When the fault appeared, the programs executed the diagnostic system to detect and isolate the faulty component.

### 4.1 Results using analytical redundancy methods

**Fault diagnosis using structured hypothesis test.** We verify the diagnostic system applying all the faulty modes with the two types of inputs. Figure 6 shows the obtained results when we introduced a fault in the damper. Top plots show: impulse signal to the system, the vehicle position and vertical speed during the perturbation. Bottom plots show: parametric identification with RLS, and the parameters values to test the hypothesis. The diagnostic system detected the fault and isolated the faulty component (the damper).

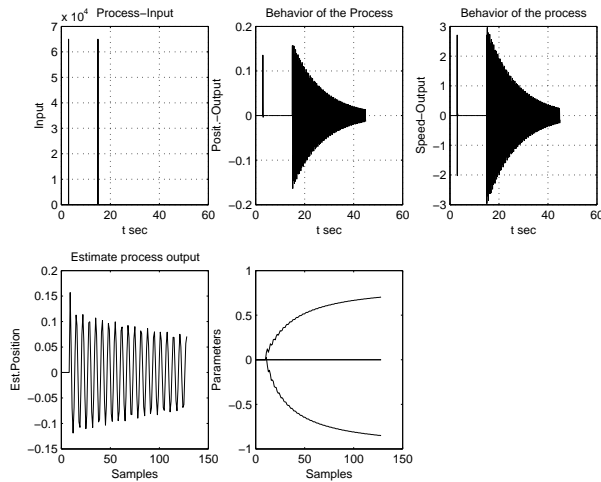


Figure 6: Faulty damper detection

**Fault diagnosis using the transfer function.** Table 3 presents the results obtained when we introduced the sinusoidal input. The tests were done with different vehicle speeds. Figure 7 shows the results obtained when we applied the oil leak in the damper. Top plots show: The perturbation to the system, the vehicle position and the vertical speed. Bottom plots show: parametric identification with RLS, the parameters of the transfer function, and the bode diagram. In this case, the diagnostic system detected the fault, but it did not isolate the faulty component. The bode diagram was computed with Matlab and it only confirms the faulty situation with the frequency response.

Table 3. Results obtained for the sinusoidal input.

Faulty Mode	Warning	Km/H	Observations
$F_S$	Verify Suspension Syst.	50	None
$F_{D1}$	Loss-damper	75	None
$F_{D2}$	Loss-damper	40	The faulty component is not detected. See Figure 7
$F_{SD1}$	Loss-damper	50	None
$F_{SD2}$	Loss-damper	40	The faulty component is not detected.

## 4.2 Diagnosis results using sequential Monte Carlo methods

To apply this method it was necessary to calculate the matrices of the discrete state spaces for each faulty mode. Then, we simulated the transition matrix to evaluate the evolution of the dynamic system over time. The following test were done:

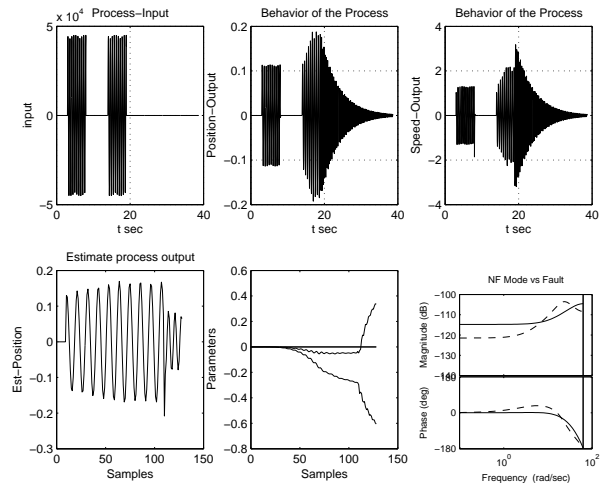


Figure 7: Oil Leak in Damper.

1. We simulated the dynamic system and some faults were implemented over time. Figure 8 presents the obtained results with the fault mode  $F_{SD1}(Z6)$ . The diagnostic system detects the fault and it isolates the faulty component very well. Top plot shows the behavior of the system with fault, and middle plot shows the identification made with the laRBPF algorithm. Bottom plot shows the faulty mode isolated by the diagnostic system. During the tests, we obtained an effectiveness of 80% of the laRBPF algorithm to detect and isolate the faulty components.

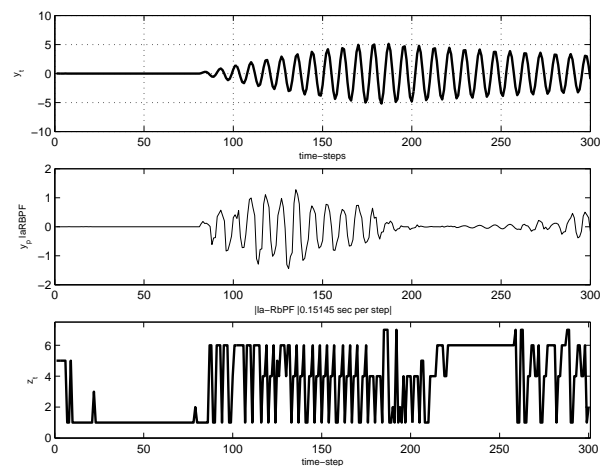


Figure 8: Results for the spring and damper faults.

2. We executed the simulation, and we applied the same faults sequence of test one, but the algorithm was evaluated with 40 particles. We can

observe, in Figure 9, that mistake was reduced to identify the fault mode, Z5, but the algorithm requires more time to get the diagnosis.

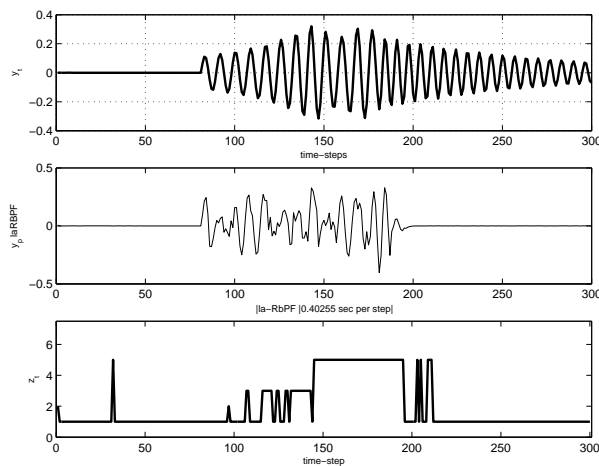


Figure 9: Results for the oil leak in the damper.

## 5 Conclusions

We have described a new general framework of a FDI system applied to a mechanical system. We modeled the suspension system using equations of the state space. Thereafter, we defined the components, faults, faulty modes and the diagnostic system to detect and isolate the fault in the mechanical system. One important part was the diagnostic system, where we established the decision rules to get the diagnostic statement in the system. We developed different techniques to diagnose the mechanical system and we demonstrated how these algorithms can detect and isolate the faults with excellent results. The present work defines a methodology to implement a diagnostic framework in any mechatronic system.

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